Communication

On The Maximum Absorbed Power in Receiving Antenna Arrays

Ha Bui-Van, Michel Arts, Christophe Craeye, *Senior Member, IEEE*, Eloy de Lera Acedo

***Abstract*—The loading condition to maximize the total received power in arbitrary antenna arrays is presented. It is found that the loads should equal the conjugate of the active impedance of the array in the transmitting case. A physical insight into the solution is provided. Numerical validation on a 2-D non-regular antenna array is provided.**

***Index Terms*—Receiving antenna array, optimum loads, received power, mutual coupling.**

I. Introduction

In receiving antenna arrays, the loads are often designed so as to maximize the power transferred to the loads from the incident wave. In [1], De Hoop has shown that the optimum impedance matrix of the receiving network should be equal to the complex conjugate of the array impedance matrix in the transmit case. To design such a solution, one however needs to realize a matrix of loads, where the loads are coupled from an electromagnetic point of view. In many systems, each antenna is connected to an isolated receiver. The antenna is thus connected directly to a single load rather than to a coupled receiving network. For such systems, a systematic approach to design/adjust the loads to maximize the received power is looked for.

In [2], the authors have estimated that these optimal loads might link to the active impedance, as could be proven for the special case of infinite periodic arrays [3]. In this Communication, we present the optimum condition for designing such loads. An analytical solution for isolated loads is derived from the case of coupled loads [1]; it requires the knowledge of the impedance matrix of the array and of the open-circuit voltages defined on receive. That solution can be applied to power transfer for both far and near-fields sources. The link with the active impedance solution used in infinite periodic arrays is established and the steps need to put this solution into practice are described. Finally, the solution is validated by comparison with solutions from a global optimization on a 2-D array of complex antennas.

The remainder of this Communication is organized as follows: in Section II, the theory on optimum loads is presented. The solution is derived and its physical meaning is discussed. Numerical validation is provided in Section III. Concluding remarks are made in Section IV.

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H. Bui-Van is with AGC Glass Europe, AGC Technovation Center, 6041 Gosselies, Belgium (email: buivanhabk@gmail.com).

M. Arts is with ASTRON, the Netherlands.

C. Craeye is with ICTEAM, Université catholique de Louvain, Belgium (email: christophe.craeye@uclouvain.be).

E. de Lera Acedo is with Cavendish Astrophysic, University of Cambridge.

II. Theory on Maximizing Received Power

Considering an array of N antennas, the array is characterized by the impedance matrix $\mathbf{Z}$. In the receiving case, the open-circuit voltage at the port of antenna m for a given incident wave can be defined as follows [4, Sec. 4.2.1], [5, eq. (49)-(51)]

$$\mathbf{v}_{\mathrm{m}}^{\mathrm{o.c.}} = \frac{1}{\mathrm{I}_{\mathrm{m}}^{\mathrm{T}}} \iint_{\mathrm{S_b}} (\vec{\mathrm{E}}^{\mathrm{I}} \times \vec{\mathrm{H}}_{\mathrm{m}}^{\mathrm{T}} - \vec{\mathrm{E}}_{\mathrm{m}}^{\mathrm{T}} \times \vec{\mathrm{H}}^{\mathrm{I}}).\hat{n}_b \, dS \tag{1}$$

with $\mathrm{S_b}$ a surface enclosing the array, $\hat{n}_b$ the (outward) unit vector normal to $\mathrm{S_b}$, $(\vec{\mathrm{E}}^{\mathrm{I}}, \vec{\mathrm{H}}^{\mathrm{I}})$ the electric and magnetic incident fields, and $(\vec{\mathrm{E}}_{\mathrm{m}}^{\mathrm{T}}, \vec{\mathrm{H}}_{\mathrm{m}}^{\mathrm{T}})$ the electric and magnetic fields radiated by the array in the transmit case when the element m is excited by a current source $\mathrm{I}_{\mathrm{m}}^{\mathrm{T}}$ while the other elements are left open. Equation (1) is valid for both near and far field incident waves. For the far-field case, where the incident wave is a plane wave, the integration (1) becomes the scalar product of incident plane wave with the open-circuit pattern [1]. In this communication, we will derive the loading condition to maximize the received power, given the information of $\mathbf{v}^{o.c.}$, including the near-field case, i.e. when the fields incident on the array present a substantially curved phase front. The only condition for application to the near-field is that the physical presence of the receiving array does not significantly affect the incident fields themselves $(\vec{\mathrm{E}}^{\mathrm{I}}, \vec{\mathrm{H}}^{\mathrm{I}})$, which means that currents on the source device are not altered (no significant multiple-scattering between source device and receiving array).

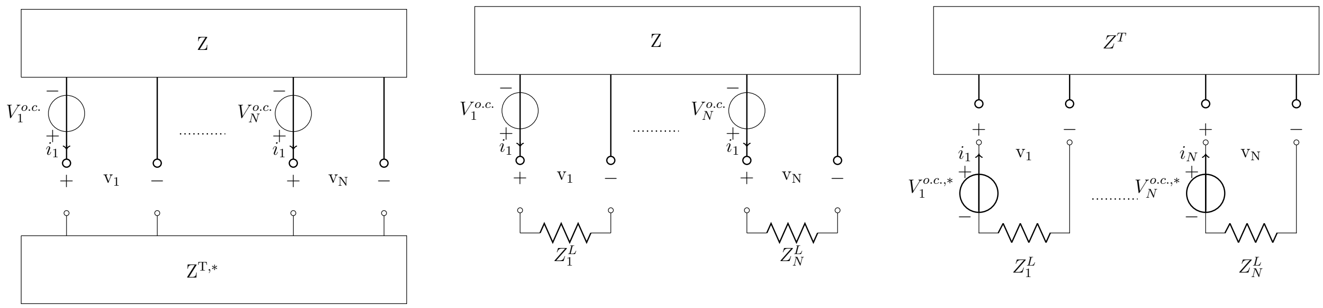
The array is then connected to a series of loads, described by the diagonal matrix $\mathbf{Z}^{\mathrm{L}}$, and its Thevenin equivalent circuit is shown in Fig.1(b) [1], [6]. Denoting $\mathbf{v}$ and $\mathbf{i}$ as the column vectors of voltages across the ports and of currents flowing through the ports, respectively, the following relations are found:

$$\mathbf{v}_{\mathrm{m}}^{\mathrm{o.c.}} = \mathrm{v}_{\mathrm{m}} + \mathbf{u}_{\mathrm{m}}^{\mathrm{T}} \mathbf{Z}^{\mathbf{T}} \, \mathbf{i} \tag{2}$$

where $\mathbf{u}_{\mathrm{m}}$ is a $N \times 1$ vector of zeros, except for a unit value at the m^{th} position, while $(.)^{\mathrm{T}}$ denotes the transpose operation. The total time-average power dissipated in the loads is obtained as

$$P^L = \frac{1}{2} \mathrm{Re} \Big(\sum_{\mathrm{m}=1}^{\mathrm{N}} \mathrm{v}_{\mathrm{m}} \mathrm{i}_{\mathrm{m}}^{*} \Big) \tag{3}$$

where $(.)^*$ denotes complex conjugation. The following section derives the optimum values of the loads for maximum received power. It makes use of the result provided by De



(a) Receive array with matrix load impedance. (b) Receive array with series load impedance. (c) Transmit array with optimum generator loads.

Fig. 1. Equivalent circuit of receiving and transmitting arrays.

Hoop [1] regarding the optimal receiving network. It goes beyond that result in that, here, a series of isolated loads attached to the ports rather than a receiving network represented by an impedance matrix. This is made possible through the knowledge of the open-circuit voltages, whose general expression has been reminded in (1). Moreover, an interpretation is provided in terms of active impedances.

A. Existence of an optimum

Following [1], to maximize the received power, the load matrix impedance of the receiving network is equal to $\mathbf{Z}^{\mathrm{T},*}$, i.e. the conjugate of the array impedance matrix in the transmit case. For this load matrix, the port voltages and currents are $\mathbf{v}$ and $\mathbf{i}$, respectively. We will now derive an equivalent series of loads $\mathbf{Z}^{\mathrm{L}}_{\mathrm{port}}$ attached to the ports that is able to produce the same vector current as the load matrix. The voltage at port m must be the same for coupled and isolated loads, it can be expressed as:

$$\mathrm{v_m} = \sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{Z}^*_{\mathrm{m,n}}\, \mathrm{i_n} = \mathrm{Z}^{\mathrm{L}}_{\mathrm{port,m}}\, \mathrm{i_m} \tag{4}$$

Dividing both sides by $\mathrm{i_m}$, the load at the port is then obtained as:

$$\mathrm{Z}^{\mathrm{L}}_{\mathrm{port,m}} = \frac{\sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{Z}^*_{\mathrm{m,n}}\, \mathrm{i_n}}{\mathrm{i_m}} = \frac{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\, \mathbf{Z}^*\mathbf{i}}{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\, \mathbf{i}} \tag{5}$$

The isolated loads in (5) depend on the knowledge of the currents in the optimal coupled receiving network, which can be traced back to array impedance matrix and open-circuit voltages. Indeed, based on the solution from [1] (shown in Fig. 1(a)), we also have:

$$\mathbf{i} = -\left(\mathbf{Z}^{T} + \mathbf{Z}^{\star}\right)^{-1}\ \mathbf{v}^{o.c.} \tag{6}$$

Hence, given the knowledge of the source, it is possible to find a set of loads producing the same port voltages as in the presence of the fully coupled optimal receiving network. Since the boundary condition at the ports level is directly given in terms of equivalent isolated load impedances, it also ensures that the currents are the same as in the solution proposed in [1]. This will allow us to also write the port current as

$$\mathbf{i} = -\left(\mathbf{Z}^{T} + \mathbf{Z}^{L}\right)^{-1}\ \mathbf{v}^{o.c.} \tag{7}$$

Therefore, the maximum power absorbed by the isolated loads must also correspond to the maximum power obtained with a coupled receiver.

B. Determination of the series load impedance

Now existence of the optimal series of load impedance is ensured, we establish an explicit expression for calculating these loads. From (5) and (6), the load impedance is obtained as

$$\mathrm{Z}^{\mathrm{L}}_{\mathrm{port,m}} = \frac{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\, \mathbf{Z}^*\mathbf{i}}{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\, \mathbf{i}} = \frac{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\mathbf{Z}^*(\mathbf{Z}^{\mathrm{T}} + \mathbf{Z}^*)^{-1}\mathbf{v}^{\mathrm{o.c.}}}{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}(\mathbf{Z}^{\mathrm{T}} + \mathbf{Z}^*)^{-1}\mathbf{v}^{\mathrm{o.c.}}} \tag{8}$$

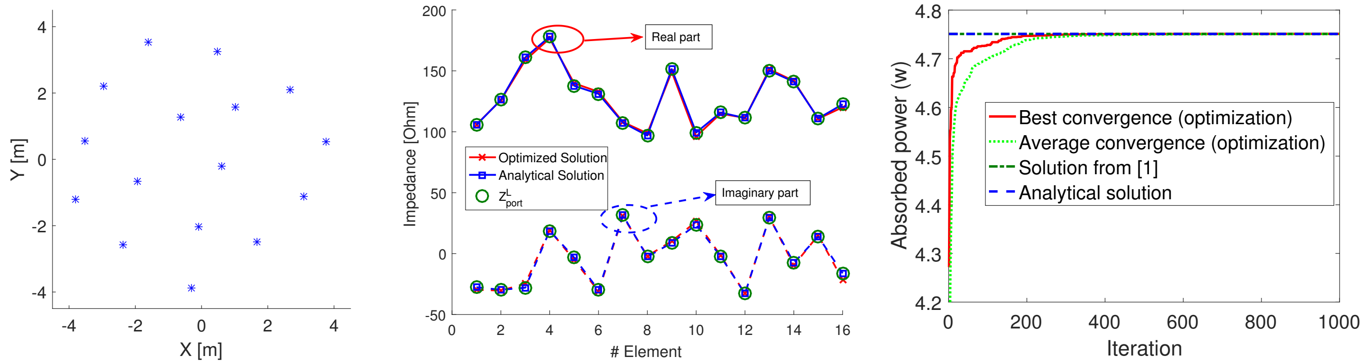
From equation (8), the values of these loads are directly obtained as soon as the open-circuit voltage is determined.

C. Interpretation of the Solution

It is known that, for infinitely periodic arrays, the optimum impedance for maximum power absorption corresponds to the conjugate of the active impedance. In this way, arrays able to absorb all the power incident from a plane wave have been designed. The active impedance corresponds to the ratio between voltage and current in the infinite periodic array when, in a transmitting configuration, it is scanned in the direction of interest, which supposed that all elements are transmitting (from there the name). For the general array referred to above, one may also define active impedances. They are obtained through a re-interpretation of the equivalent circuit of the receiving array and its termination as that of a transmitting array, shown in Fig. 1(c). In this case, what used to correspond to open-circuit voltages of the receiving arrays now, after conjugation, corresponds to the open-circuit voltages of the different generators. The use of voltages equal to the conjugates of the open-circuit voltages on receive ensure scanning in the direction of the incident wave, or time-reversal for the more general case of near-field sources (see the theory and implementation of focusing through time reversal [8], [9]). Moreover, the generator loads are designed as Z^H (with ^{H} being the Hermitian operator) as it would allow maximum power deliver to the antenna (see appendix of [1]).

From Fig. 1(c), the active impedance with $\mathbf{v}^{\mathrm{o.c.},*}$ voltage source is written as:

$$\mathrm{Z}^{\mathrm{act}}_{\mathrm{m}} = \frac{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\mathbf{Z}(\mathbf{Z} + \mathbf{Z}^{\mathrm{H}})^{-1}\mathbf{v}^{\mathrm{o.c.},*}}{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}(\mathbf{Z} + \mathbf{Z}^{\mathrm{H}})^{-1}\mathbf{v}^{\mathrm{o.c.},*}} \tag{9}$$



(a) Configuration of 16 SKALA array. (b) Real and imaginary parts of optimum loads. (c) Convergence curve of GSO over 30 trials.

Fig. 2. Optimization of loads to maximize the received power of a 16-element array for the incident plane wave from direction (θ_0, ϕ_0) = $(90°, 30°)$.

From (8) and (9), it is possible to show that

$$Z_{\mathrm{m}}^{\mathrm{L,o}} = Z_{\mathrm{m}}^{\mathrm{act},*} \tag{10}$$

This configuration is also consistent with the classical solution referred to above for infinite periodic arrays.

One also can replace the coupled generator loads by isolated loads using expression follows:

$$Z_{\mathrm{m}}^{\mathrm{trans}} = \frac{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\, \mathbf{Z}^{\mathrm{H}} \mathbf{I}}{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}}\, \mathbf{I}} = \frac{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}} \mathbf{Z}^{\mathrm{H}} (\mathbf{Z} + \mathbf{Z}^{\mathrm{H}})^{-1} \mathbf{v}^{\mathrm{o.c.},*}}{\mathbf{u}_{\mathrm{m}}^{\mathrm{T}} (\mathbf{Z} + \mathbf{Z}^{\mathrm{H}})^{-1} \mathbf{v}^{\mathrm{o.c.},*}} \tag{11}$$

D. Implementation

In practice, to make this solution possible, one needs a method for determining the open-circuit voltages and a means to tune the impedance of each independent receiver. The former may be achieved through numerical sumulation, while, in a narrow-band context, a different matching circuit may need to be introduced between the antennas and each receiver. The design of such a circuit it outside the scope of this paper.

III. Validation

The analytical solution proposed in Section II is now applied to maximize the power received of an array of 16 SKALA (SKA Log-periodic Antennas), which have been proposed for the SKA low-frequency project [10]. In the example below, one of the two polarizations (along x) of each antenna is excited. The incident plane wave is assumed to be coming from the (θ_0, ϕ_0) = $(90°, 30°)$ direction and to have the same polarization as the co-polarization of the array (θ and ϕ are elevation and azimuth angles, respectively). To validate the result described in Section II-B, a global optimization is applied to optimize the loads. For the optimization, the Genetical Swarm Optimization (GSO) technique [11] is implemented. The fitness function maximizes the power defined in (3) by controlling the imaginary and real parts of the loads. As for GSO's parameters, a population of 100 is set for the optimization, which is shared equally for Genetic algorithm (GA) and Particle swarm optimization (PSO) kernels in the GSO. In this problem with infinite plane wave excitation, another validation considers the solution of [1] using a load matrix. For the analytical approach, series loads are directly obtaine from eq. (8) for given incident wave. To fully account for mutual coupling, the simulation technique HARP proposed in [12] is exploited to simulate the array and calculate all the open-circuit patterns. The array is analyzed at 110 MHz.

Fig. 2(a) shows the configuration of the array, while the optimized loads are plotted in Fig. 2(b). The loads optimized using GSO almost overlap those from the analytical solution. A maximum difference of 3 Ω is observed, which leads to a slightly lower absorbed power using the solution of GSO, i.e. a relative deviation in terms of power smaller than 0.001 w.r.t. the analytical solution. Due to the stochastic nature of the GSO, in different trials, GSO has converged to different values, which are extremely close. These values are smaller than the one obtained using analytical solution. Fig. 2(c) plots the convergence of GSO over 30 independent trials, where the GSO converged to the optimum value of the analytical solution, which is the same as the case of load matrix (see Fig. 2(c)). This example validates the solution proposed in this Communication.

IV. Conclusion

We have presented the optimum condition for the loads in receiving arrays. It corresponds to the conjugate of the active impedance of the array scanned in the direction of incidence. This generalizes the solution proposed in [3] to non-regular (and actually arbitrary) arrays. Moreover, we also linked the proposed solution to the one given in [1] for a fully coupled receiver. We showed that those two optimum conditions coincide in terms of absorbed power, with the essential difference that the solution proposed for the isolated loads depends on the incident wave, through the open-circuit voltages. The obtained condition is validated on a 2-D array of complex antennas, where the solution of a global optimization converges to the proposed one.

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